14) $$\lim_{\theta \to \frac{\pi}{2}} \frac{1-\sin\theta}{\csc\theta} = 0$$ L'H

$$\frac{1}{\sin\theta}$$

18) $$\lim_{x \to \infty} \frac{\ln(\ln x)}{x} = \lim_{x \to \infty} \frac{\frac{1}{\ln x}\left(\frac{1}{x}\right)}{1}$$

$$\lim_{x \to \infty} \frac{1}{x \ln x} = 0$$

34) $\lim_{x\to\infty} \frac{(x^2+2)^{1/2}}{(2x^2+1)^{1/2}} = \lim_{x\to\infty} \frac{\frac{1}{2}(x^2+2)^{-1/2}(2x)}{\frac{1}{2}(2x^2+1)^{-1/2}(4x)}$

$= \lim_{x\to\infty} \frac{(2x^2+1)^{1/2}}{2\,(x^2+2)^{1/2}} = \lim_{x\to\infty} \frac{\frac{1}{2}(2x^2+1)^{-1/2}(4x)}{\frac{1}{2}(2)(x^2+1)^{-1/2}(2x)}$

$= \lim_{x\to\infty} \frac{(x^2+1)^{1/2}}{(2x^2+1)^{1/2}}$

$$\lim_{x\to\infty} \frac{(x^2+2)^{1/2}}{(2x^2+1)^{1/2}} = \lim \frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}}$$

$$= \lim \frac{\sqrt{x^2}\sqrt{1+\frac{2}{x^2}}}{\sqrt{x^2}\sqrt{2+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{\sqrt{2+\frac{1}{x^2}}} =$$

$$\boxed{\frac{\sqrt{1}}{\sqrt{2}}}$$

" ∞^0 "

$$\lim_{x\to\infty} x^{\left(\frac{1}{x}\right)}$$

$x=100$

$100^{\frac{1}{100}}$

≈ 1.047

$x=120$ in base

$120^{\frac{1}{100}}$

$1.049 \Uparrow$

$x=120$ in exp

$100^{\frac{1}{120}}$

$1.0391 \Downarrow$

" 0^∞ " NOT INDETERMINATE

$$\lim_{x\to 0} x^{\left(\frac{1}{x}\right)}$$

$x=.6$

$(.6)^{\left(\frac{1}{.6}\right)}$

$.4268$

$x=.5$ in base

$(.5)^{\left(\frac{1}{.6}\right)}$

$.315 \Downarrow$

$x=.5$ in exp

$(.6)^{\left(\frac{1}{.5}\right)}$

$.36 \Downarrow$

how to use L'H to resolve limits of form "∞^0"

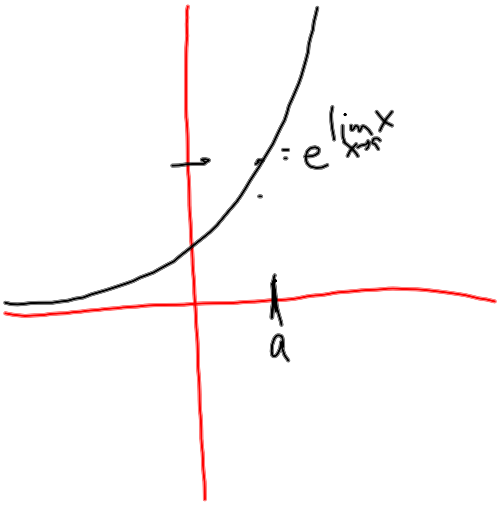
$$\lim_{x\to\infty} x^{\left(\frac{1}{x}\right)} = \lim_{x\to\infty} e^{\ln\left(x^{\frac{1}{x}}\right)} = e^{\lim_{x\to\infty} \ln\left(x^{\frac{1}{x}}\right)}$$

$$= e^{\lim_{x\to\infty} \left(\frac{1}{x}\right)(\ln x)}$$

$$= e^{\lim_{x\to\infty} \left(\frac{\ln x}{x}\right)} = e^{\lim_{x\to\infty} \frac{\frac{1}{x}}{1}} = e^0 = 1$$

$\ln a^b = b \ln a$

$e^{\ln a} = a$



58)

$$\lim_{x\to\infty} x^{\left(\frac{\ln 2}{1+\ln x}\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{\ln 2}{1+\ln x}\right)\ln x}$$

$$= e^{\lim_{x\to\infty}\frac{(\ln x)(\ln 2)}{1+\ln x}} = e^{\lim_{x\to\infty}\frac{\frac{\ln 2}{\cancel{x}}\cancel{(x)}}{\frac{1}{\cancel{x}}\cancel{(x)}}} = e^{\lim_{x\to\infty}\ln 2}$$

$$= e^{\ln 2} = 2$$

Apply L'H rule

$\infty - \infty$

Goal: turn into fraction

Strategy:

* common denominator

OR

* multiply by conjugate

$0 \cdot \infty$

Goal: turn it into fraction

strat: consider EITHER

$\frac{\infty}{\left(\frac{1}{0}\right)}$ or $\frac{0}{\left(\frac{1}{\infty}\right)}$

0^0

Goal: turn into $(0 \cdot \infty)$

strategy: consider

$e^{\ln(0^0)}$

1^∞ ∞^0

Goal: turn into $(0 \cdot \infty)$

strategy: consider

$e^{\ln(1^\infty)}$

HW) sheet

39, 40, 44, 47, 50, 53–56, 60, 63

39) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right)$

53) $\lim_{x\to 0^+} x^{(x^2)}$

47) $\lim_{x\to 1} \frac{x}{x-1} - \frac{1}{\ln x}$

55) $\lim_{x\to 0} (1-2x)^{\frac{1}{x}}$

39) $\lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\cos\left(\frac{\pi}{x}\right)\left(-\frac{\pi}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = \lim_{x\to\infty} \pi\cos\left(\frac{\pi}{x}\right)$

$= \pi(1) = \pi$

34)

$$\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}} = \lim_{x\to\infty}\frac{(x^2+2)^{1/2}}{(2x^2+1)^{1/2}} = \lim_{x\to\infty}\frac{\frac{1}{2}(x^2+2)^{-1/2}(2x)}{\frac{1}{2}(2x^2+1)^{-1/2}(4x)}$$

$$= \lim_{x\to\infty}\frac{(x^2+2)^{-1/2}}{2(2x^2+1)^{-1/2}} = \lim_{x\to\infty}\frac{(2x^2+1)^{1/2}}{2(x^2+2)^{1/2}}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{2}(2x^2+1)^{-1/2}(4x)}{\cancel{(\frac{1}{2})}\cancel{(2)}(x^2+2)^{-1/2}(2x)} = \lim_{x\to\infty}\frac{(x^2+2)^{1/2}}{(2x^2+1)^{1/2}}$$

34) $\lim_{x\to\infty} \dfrac{\sqrt{x^2+2}}{\sqrt{2x^2+1}} = \lim_{x\to\infty} \dfrac{\sqrt{x^2}\sqrt{1+\frac{2}{x^2}}}{\sqrt{x^2}\sqrt{2+\frac{1}{x^2}}}$

$= \lim_{x\to\infty} \dfrac{\sqrt{1+\frac{2}{x^2}}}{\sqrt{2+\frac{1}{x^2}}} = \sqrt{\frac{1}{2}}$

Recall $\lim_{x \to \infty} \frac{x^2}{x+4}$

Set $x = 100$

set $x = 120$ in the num

set $x = 120$ in the denom

"∞^0" IS I.F.

$$\lim_{x\to\infty} x^{\left(\frac{1}{x}\right)}$$

$$=\lim_{x\to\infty} e^{\ln\left(x^{\frac{1}{x}}\right)}$$

$$=\lim_{x\to\infty} e^{\left(\frac{1}{x}\ln x\right)}$$

$$=e^{\left(\lim_{x\to\infty}\frac{1}{x}\ln x\right)}$$

$$=e^{\left(\lim_{x\to\infty}\frac{\ln x}{x}\right)}$$

$$=e^{\lim_{x\to\infty}\left(\frac{\frac{1}{x}}{1}\right)}=e^0=1$$

$x=10$

$10^{1/10}=1.2589$

$x=20$ in base

$20^{1/10}=1.34 \Uparrow$

$x=20$ in exponent

$10^{1/20}=1.12 \Downarrow$

"0^∞" Not I.F.

$$\lim_{x\to 0} x^{\frac{1}{x}}$$

$x=.6$

$(.6)^{\frac{1}{.6}}=.426$

$x=.5$ in base

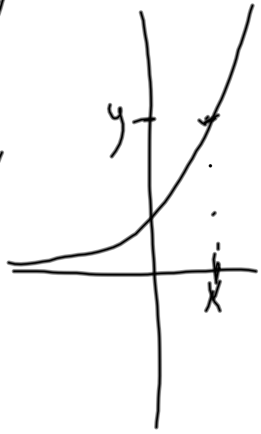
$.5^{\frac{1}{.6}}=.314 \Downarrow$

$x=.5$ in exp

$(.6)^{\frac{1}{.5}}=.36 \Downarrow$

$\ln a^b = b\ln a$

$e^{\ln a}=a$



$$\lim_{x\to 4} e^x = e^4$$

$$e^{\lim_{x\to 4} x} = e^4$$

58) $\lim_{x \to \infty} x^{\left(\frac{\ln 2}{1+\ln x}\right)}$

$$= e^{\lim_{x\to\infty} \left(\frac{\ln 2}{1+\ln x}\right)(\ln x)} = e^{\lim_{x\to\infty} \frac{(\ln 2)(\ln x)}{1+\ln x}} = e^{\lim_{x\to\infty} \frac{(\ln 2)\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}} =$$

$$e^{\lim_{x\to\infty} \ln 2} = e^{\ln 2} = 2$$

To apply L'H Rule - -

indeterminate form:	"$0 \cdot \infty$"	"$\infty - \infty$"	0^0
goal:	turn into a fraction	turn into a fraction	turn into $0 \cdot \infty$
strategy:	$\frac{\infty}{\left(\frac{1}{0}\right)}$ or $\frac{0}{\left(\frac{1}{\infty}\right)}$ $\Downarrow$ L'H $\Downarrow$ L'H	* common denominator * mult. by conjugate	consider $e^{\ln(0^0)}$ ⋮